Jump filtering and efficient drift estimation for Lévy-driven SDE's

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Model I

Let $\Theta \subset \mathbb{R}^d$, Θ-compact. We aim at estimating the unknown drift parameter $\theta \in \Theta$ of a jump diffusion process X^θ given by

$$X_t^\theta = X_0^\theta + \int_0^t b(\theta, X_s^\theta)\, ds + \int_0^t \sigma(X_s^\theta)\, dW_s + \int_0^t \gamma(X_{s-}^\theta)\, dL_s$$

where $t \in \mathbb{R}_+$, $W = (W_t)_{t \geq 0}$ is a one-dimensional Brownian motion and L a pure jump Lévy process with Lévy measure ν, such that

$$\int_{\{0<|z|\leq 1\}} |z| \nu(dz) < \infty.$$

Sampling scheme

High frequency data with an observation time going to infinity:

$$0 \leq t_0 \leq \ldots \leq t_n \qquad X^\theta_{t_0}, \ldots, X^\theta_{t_n}$$

such that

$$\Delta_n := max\{t_i - t_{i-1} : 1 \leq i \leq n\} \to 0, \qquad \text{as} \quad n \to \infty;$$

$$t_n \to \infty \qquad \text{and} \qquad t_n = O(n\Delta_n).$$

Goals:

- efficient estimation of the drift parameter,
- minimal conditions on the sampling step Δ_n.

Literature about the high frequency inference for diffusion with jumps

- [Masuda (13)]: Gaussian quasi-likelihood estimators
- [Shimizu and Yoshida (06)]: contrast-type estimation function, jumps of compound Poisson type.
- [Shimizu (06)]: include more general driving Lévy processes.
- [Tran(14)]: LAN property for drift and diffusion parameters via Malliavin calculus.
- [Mai(2014)]: drift estimation for Lévy-driven Ornstein-Uhlenbeck.

except [Mai(2014)], joint estimation of the drift, diffusion and jump part parameters is considered;
under condition which is at best

$$n\Delta_n^2 \to 0.$$

- The estimation of the volatility is feasible on a compact interval,
- the estimation of the drift and the jump law requires a growing time window.
- Due to the Poisson structure of the jump part the estimation of the jump parameter can be well separated from those of the drift and the diffusion.

We focus on the estimation of the drift parameter only; and construct a consistent, asymptotically normal and efficient estimator, under conditions

$$n\Delta_n^{3-\varepsilon} \to 0.$$

Remark: the condition $n\Delta_n^3 \to 0$ was found by [Florens-Zimrou(89)] and [Yoshida(92)] in the case of drift estimation for continuous diffusions.

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The equation of the model can be rewritten as

$$X_t^\theta = X_0^\theta + \int_0^t b(\theta, X_s^\theta)\, ds + \int_0^t \sigma(X_s^\theta)\, dW_s + \int_0^t \int_{\mathbb{R}} \gamma(X_{s-}^\theta) z \mu(ds, dz)$$

where μ is the Poisson random measure on $[0, \infty) \times \mathbb{R}$,

$$L_t = \int_0^t \int_{\mathbb{R}} z \mu(ds, dz)$$

is the Lévy process with Lévy-Khintchine triplet $(0, 0, \nu)$ such that

$$\int_{\{0<|z|\leq 1\}} |z| d\nu(z) < \infty.$$

X_0^θ, W and L are independent.

Assumption (Existence)

Assumption (Irreducibility)

Assumption (Non-degeneracy)
There exists some $\alpha > 0$, *such that* $\sigma^2(x) \geq \alpha$ *for all* $x \in \mathbb{R}$.

Assumption (Identifiability)

Assumption (Hölder-continuity of the drift and its 1,2 derivatives with respect to θ.)

Assumption (Subpolynomial growth of all Hölder constants)

Assumption (Jumps)

The jump coefficient γ is bounded from below;
If $\nu(\mathbb{R}) = \infty$, $\int_{0<|z|\leq 1} |z|\nu(dz) < \infty$, the Lévy measure ν is absolutely continuous with respect to the Lebesgue measure, and γ is upper bounded.

Assumption (Ergodicity)

(i) *For all $q > 0$, $\int_{|z|>1} |z|^q \nu(dz) < \infty$.*

(ii) *For all $\theta \in \Theta$ there exists a constant $C > 0$ such that $xb(\theta, x) \leq -C|x|^2$, if $|x| \to \infty$.*

(iii) $|\gamma(x)|/|x| \to 0$ *as* $|x| \to \infty$.

(iv) $|\sigma(x)|/|x| \to 0$ *as* $|x| \to \infty$.

(v) $\forall \theta \in \Theta$, $\forall q > 0$ *we have* $\mathbb{E}|X_0^\theta|^q < \infty$.

The last Assumption ensure the existence of unique invariant distribution π^θ, as well as the ergodicity of the process X^θ, similarly to [Masuda(2007)].

Lemma

For all $\theta \in \Theta$, X^θ admits a unique invariant distribution π^θ and the ergodic theorem holds:

1. *for every measurable function $g : \mathbb{R} \to \mathbb{R}$ satisfying $\pi^\theta(g) < \infty$, we have $a.s.$*

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t g(X_s^\theta) ds = \pi^\theta(g).$$

2. *For all $q > 0$, $\pi^\theta(|x|^q) < \infty$.*
3. *For all $q > 0$, $\sup_{t\in\mathbb{R}} E[|X_t^\theta|^q] < \infty$.*
4. *Moreover,*

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t E[|X_s^\theta|^q] ds = \pi^\theta(|x|^q).$$

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Construction of the estimator

Recall that we observe a finite sample

$$X_{t_0}, \ldots, X_{t_n}; \quad 0 = t_0 \leq t_1 \leq \ldots \leq t_n \tag{1}$$

A natural approach to estimate the unknown drift parameter =MLE

- the likelihood function based on the discrete sample is not tractable in this setting, since it depends on the transition densities of X which are not explicitly known.
- On the contrary, the continuous-time likelihood function is explicit.

Denote the true parameter value by $\theta^\star$, We shorten X for $X^{\theta^\star}$ and P, E, π for respectively $P^{\theta^\star}, E^{\theta^\star}, \pi^{\theta^\star}$.
The continuous time likelihood function is given by

$$\mathcal{L}_t(\theta, X) = \frac{dP_t^\theta}{dP_t}(X) =$$
$$\exp\left(\int_0^t \sigma(X_s)^{-2} b(\theta, X_s)\, dX_s^c - \frac{1}{2}\int_0^t \sigma(X_s)^{-2} b(\theta, X_s)^2\, ds\right).$$

We define the log-likelihood function as

$$\ell_t(\theta) := \ln \mathcal{L}_t(\theta, X).$$

Our aim is to approximate $\ell_t(\theta)$ from discrete sample and thus define some contrast.

The problem is that X^c is unobserved !

Define the increment's operator Δ_i^n :

$$\Delta_i^n X = X_{t_i} - X_{t_{i-1}}, \quad \Delta_i^n X^c = X^c_{t_i} - X^c_{t_{i-1}} \quad \Delta_i^n Id = t_i - t_{i-1}.$$

Let $(a_n^i), i = 1, \dots n$, be a sequence of positives random variables, a_n^i measurable with respect to $\{X_{t_j}; j < i\}$. We suppose there exist $\underline{a} \in \mathbb{R}_+^*$, $\bar{a} \in \mathbb{R}_+^*$ such that $0 < \underline{a} \leq a_n^i \leq \bar{a} < \infty$.
Let $\varepsilon \in (0, 1/2)$ and denote

$$v_n^i = a_n^i v_n, \quad v_n = \Delta_n^{1/2-\varepsilon}, n \geq 1,\ i = 1 \dots, n. \qquad (2)$$

$$\ell_{t_n}^n(\theta) = \sum_{i=1}^n \frac{b(\theta, X_{t_{i-1}})}{\sigma(X_{t_{i-1}})^2} \Delta_i^n X \mathbf{1}_{\{|\Delta_i^n X| \leq v_n^i\}} - \frac{1}{2} \sum_{i=1}^n \frac{b(\theta, X_{t_{i-1}})^2}{\sigma(X_{t_{i-1}})^2} \Delta_i^n Id.$$

Finally, we define an estimator $\hat{\theta}_n$ of the true value $\theta^\star$ as

$$\hat{\theta}_n \in \underset{\theta \in \Theta}{\operatorname{argmax}} \, \ell_{t_n}^n(\theta)$$

and in the sequel we call it the filtered MLE (FMLE).
This kind of thresholding technics was already used in [Shimizu and Yoshida (06)], [Mai(2014)], [Cecilia Mancini (11)].

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Main results

without further assumptions on n and Δ_n,

Théorème (Consistency)

The FMLE $\hat{\theta}_n$ is consistent in probability:

$$\hat{\theta}_n \xrightarrow{P} \theta^\star, \qquad n \to \infty.$$

Define the asymptotic Fisher information by

$$I(\theta) = \left(\int_{\mathbb{R}} \frac{\partial_{\theta_i} b(\theta, x) \partial_{\theta_j} b(\theta, x)}{\sigma^2(x)} \pi^\theta(dx) \right)_{1 \le i,j \le d}. \tag{3}$$

Assumption

For all $\theta \in \Theta$, $I(\theta)$ *is non-degenerated.*

Théorème (Asymptotic normality)

Assume furthermore that $n\Delta_n^{3-\varepsilon} \to 0$,

$$\sqrt{n}\Delta_n^{3/2-2\varepsilon}\left(\int_{|z|\geq 3\underline{a}v_n/\gamma_{min}} \nu(dz)\right)^{1-\varepsilon/2} \to 0$$

and

$$\sqrt{n\Delta_n}\left(\int_{|z|\leq 3\bar{a}v_n} |z|\nu(dz)\right)^{1-\varepsilon/2} \to 0$$

as $n \to \infty$. *Then the FMLE* $\hat{\theta}_n$ *is asymptotically normal:*

$$t_n^{1/2}(\hat{\theta}_n - \theta^\star) \xrightarrow{\mathcal{L}} N(0, I^{-1}(\theta^\star)), \quad n \to \infty.$$

The FMLE $\hat{\theta}_n$ *is asymptotically efficient.*

Remarque

If ν has a bounded Lebesgue density, all the conditions reduce to $n\Delta_n^{3-4\varepsilon} \to 0$.

Example (tempered stable jumps)

The Lévy measure in this case has an unbounded and non-integrable density given by

$$\nu(dz) = C|z|^{-(1+\alpha)}e^{-\lambda|z|}dz$$

with $\lambda > 0$ and $C > 0$ satisfies the conditions of the previous Theorem if $0 < \alpha < 1$.

The conditions on n, Δ_n and ν can now be summarized as $n\Delta_n^{2-\alpha-\tilde{\epsilon}} \to 0$ for some $\epsilon > 0$. We observe that a higher Blumenthal-Getoor index α requires a faster convergence Δ_n to zero. This is in line with the intuition that when the intensity of small jumps increases (i.e. α increases) more and more frequent observations are needed to have a sufficient performance of the jump filter.

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Jump filtering

Proposition (jump filtering)

(i) *without any assumption on the way that* $\Delta_n \to 0$ *as* $n \to \infty$,

$$\frac{1}{n\Delta_n} \sup_{\theta \in \Theta} \left| \int_0^{t_n} f(\theta, X_s)\, dX_s^c - \sum_{i=1}^n f(\theta, X_{t_{i-1}}) \Delta_i^n X \mathbf{1}_{|\Delta_i^n X| \le v_n^i} \right| \xrightarrow{P} 0;$$

(ii) *if* $n\Delta_n^{3-\varepsilon} \to 0$, $\sqrt{n}\Delta_n^{3/2-\varepsilon} \left(\int_{|z| \ge 3\underline{a} v_n/\gamma_{min}} \nu(dz) \right)^{1-\varepsilon/2} \to 0$ *and* $\sqrt{n\Delta_n} \left(\int_{|z| \le 3\bar{a} v_n} |z| \nu(dz) \right)^{1-\varepsilon/2} \to 0$, *then*

$$\frac{1}{\sqrt{n\Delta_n}} \left| \int_0^{t_n} f(\theta, X_s)\, dX_s^c - \sum_{i=1}^n f(\theta, X_{t_{i-1}}) \Delta_i^n X \mathbf{1}_{|\Delta_i^n X| \le v_n^i} \right| \xrightarrow{P} 0.$$

Lemma (Euler scheme)

(i) *as* $n \to \infty$,

$$\sup_{\theta \in \Theta} \frac{1}{n\Delta_n} \left| \int_0^{t_n} f(\theta, X_s)\, dX_s^c - \sum_{i=1}^{n} f(\theta, X_{t_{i-1}}) \Delta_i^n X^c \right| \xrightarrow{P} 0;$$

(ii) *if* $n\Delta_n^{3-\varepsilon} \to 0$, *then, as* $n \to \infty$, $\forall \theta \in \Theta$

$$\frac{1}{\sqrt{n\Delta_n}} \left| \int_0^{t_n} f(\theta, X_s)\, dX_s^c - \sum_{i=1}^{n} f(\theta, X_{t_{i-1}}) \Delta_i^n X^c \right| \xrightarrow{P} 0.$$

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Ornstein-Uhlenbeck-type processes

Suppose that we are given a discrete sample

$$X_{t_0}, \ldots, X_{t_n} \quad \text{for } t_i = i\Delta_n \text{ and } i = 0, \ldots, n, \tag{4}$$

of an Ornstein-Uhlenbeck-type (OU) process $(X_t)_{t\geq 0}$:

$$dX_t = (\theta_2 - \theta_1 X_t)\, dt + \sigma\, dW_t + dL_t \quad X_0 = x, \tag{5}$$

Our goal is to estimate the unknown drift parameter $\theta = (\theta_1, \theta_2) \in \mathbb{R}^2$. The volatility parameter $\sigma > 0$ might be unknown. The jump component $(L_t)_{t\geq 0}$ will be of compound Poisson type:

$$L_t = \sum_{i=1}^{N_t} Z_i, \text{ for } t \geq 0.$$

with intensity λ and the jump heights Z_i i.i.d. $\mathcal{N}(0,1)$.

FMLE for θ is the solution $\hat{\theta}_n^{\mathsf{OU}} = (\hat{\theta}_{1,n}^{\mathsf{OU}}, \hat{\theta}_{2,n}^{\mathsf{OU}})$ to the following set of linear equations in θ_1 and θ_2.

$$
\begin{aligned}
\theta_1 &= \frac{\theta_2 I_n(X,1) - \sum_{i=1}^n X_{t_{i-1}} \Delta_i^n X \mathbf{1}_{|\Delta_i^n X| \le v_n^i}}{I_n(X,2)}, \\
\theta_2 &= \frac{\sum_{i=1}^n \Delta_i^n X \mathbf{1}_{|\Delta_i^n X| \le v_n^i} + \theta_1 I_n(X,1)}{t_n},
\end{aligned} \tag{6}
$$

where we introduced the functional

$$
I_n(X,p) := \sum_{i=1}^n X_{t_{i-1}}^p \Delta_i^n Id \quad \text{for } p \in \mathbb{R}. \tag{7}
$$

The FLME for the first component of θ results in

$$
\hat{\theta}_{1,n}^{\mathsf{OU}} = \left(1 - \frac{I_n(X,1)^2}{t_n I_n(X,2)}\right)^{-1} \times
$$

$$
\times \frac{I_n(X,1) \sum_{i=1}^n \Delta_i^n X \mathbf{1}_{|\Delta_i^n X| \le v_n^i} - t_n \sum_{i=1}^n X_{t_{i-1}} \Delta_i^n X \mathbf{1}_{|\Delta_i^n X| \le v_n^i}}{t_n I_n(X,2)}.
$$

The second component $\hat{\theta}_{2,n}^{\mathsf{OU}}$ follows now easily by plugging $\hat{\theta}_{1,n}^{\mathsf{OU}}$ into (6).

Suppose $\theta_2 = 0$. Choose $v_n = \Delta_n^{1/2-\varepsilon} = \Delta_n^{0.49}$ and consider first the choice of constant weights $a_n^i = 5$ in the jump filter

$$\text{"decide no jump on}[t_i, t_{i+1}[\longleftrightarrow |\Delta_i^n X| < a_n^i v_n.\text{"}$$

The results of the simulations are given in columns 3–5 for $\sigma = 1$:

		$a_n^i = 5$			$a_n^i = 5 \times \widehat{\sigma_n^i}$		
t_n	n	mean	std dev	jumps filt	mean	std dev	jumps filt
10	100	1.84	0.47	1.23	1.90	0.52	0.43
	400	2.00	0.45	4.16	2.05	0.52	3.04
	1000	2.06	0.45	5.98	2.08	0.49	5.28
50	500	1.77	0.22	6.19	1.80	0.24	3.00
	2000	1.95	0.20	20.8	1.95	0.23	16.3
	5000	1.99	0.20	30.0	2.00	0.22	27.0
100	3000	1.91	0.14	34.9	1.93	0.17	25.9
	10^4	1.98	0.14	60.1	1.98	0.16	54.0
	3×10^4	2.00	0.14	76.2	2.00	0.15	73.3

Table : Monte Carlo estimates of mean and standard deviation from 5000 samples of $\hat{\theta}_{1,n}^{\mathsf{OU}}$ for an OU process with compound Poisson jumps with intensity $\lambda = 1$, $\sigma = 1$ and true parameter $\theta_1 = 2$.

If $\sigma = 3$, the same estimator appears almost useless (see columns 3–4 of Table 29). This comes from the fact that many increments of the Brownian part $\sigma(W_{t_i} - W_{t_{i-1}})$ are larger than the threshold $v_n^i = 5 \times \Delta_n^{0.49}$ in the situation $\sigma = 3$ and are confounded with jumps.

		$a_n^i = 5$			$a_n^i = 5 \times \widehat{\sigma_n^i}$		
t_n	n	mean	std dev	jumps filt	mean	std dev	jumps filt
10	100	1.33	0.51	9.16	1.94	0.59	8.4×10^{-3}
	400	1.41	0.53	35.8	2.10	0.64	0.27
	1000	1.44	0.54	85.0	2.14	0.65	1.31
50	500	1.26	0.23	45.4	1.80	0.24	0.06
	2000	1.33	0.30	180	1.98	0.28	1.53
	5000	1.35	0.23	425.0	2.01	0.28	6.63
100	3000	1.30	0.16	273	1.95	0.19	0.11
	10^4	1.34	0.17	850	1.99	0.19	13.2
	3×10^4	1.36	0.16	2386	2.01	0.19	36.4

Monte Carlo estimates of mean and standard deviation from 5000 samples of $\hat{\theta}_{1,n}^{\mathsf{OU}}$ for an OU process with compound Poisson jumps with intensity $\lambda = 1$, $\sigma = 3$ and true parameter $\theta_1 = 2$.

Hence, it is important for finite sample properties of the estimator to take into account the volatility of X^c for the choice of the jump threshold.
We introduce the threshold $v_n^i = 5 \times \widehat{\sigma_n^i} \times \Delta_n^{0.49}$, where $(\widehat{\sigma_n^i})^2$ is an estimation of the quadratic variation of the process on $K = 30$ past observations,

$$(\widehat{\sigma_n^i})^2 = \frac{1}{\Delta_n K} \sum_{l=1}^{K} \left(\Delta_{i-l}^n X\right)^2 \tag{8}$$

and for convenience we set $(\widehat{\sigma_n^i})^2 = (\widehat{\sigma_n^{K+1}})^2$ for $1 \leq i \leq K$.

Remark: the number of filtered jumps is much smaller than the true expected number of jumps.
As the number of 'filtered jumps' is a decreasing function of the threshold, it is possible to find the threshold $v_i^n = a(\Delta_n)^{0.49}$, $a > 0$, such as the average number of 'filtered jumps' is equal to the expected number of jumps λt_n. It appears that the estimator has a higher bias than when the number of jumps was underestimated. Hence, it seems preferable, in some situations, to filter less jumps than the true number of jumps.

			$v_n^i = a\Delta_n^{0.49}$		
t_n	n	a	mean	std dev	jumps filt
10	400	2.551	1.88	0.41	10.0
50	5000	2.897	1.94	0.19	50.0
100	10^4	2.9	1.93	0.14	100

Table : Monte Carlo estimates of mean and standard deviation from 5000 samples of $\hat{\theta}_{1,n}^{\mathrm{OU}}$ for an OU process with compound Poisson jumps with intensity $\lambda = 1$, $\sigma = 1$, true parameter $\theta_1 = 2$, and with threshold such as the estimated number of jumps is unbiased.

Infinite activity

We consider again the O.U. model (5), with $\theta_2 = 0$, and where the driving Lévy process $(L_t)_{t\geq 0}$ is a tempered α-stable process:

$$\nu(dx) = \kappa |x|^{-(1+\alpha)} e^{-b|x|} dx,$$

for $\alpha \in (0,1)$, $b > 0$, $\kappa = \kappa_\alpha := \left[\frac{\Gamma(1-\alpha)}{\alpha} \cos(\frac{\pi}{2}\alpha)\right]^{-1}$,
which is the scale constant such that the symmetric stable process $(L_t^\alpha)_t$ with jump intensity $\kappa_\alpha |x|^{-(1+\alpha)} dx$ admits a Lévy Kintchine exponent $E[e^{iuL_1^\alpha}] = e^{-|u|^\alpha}$.
For the simulation of the increments of the tempered stable process we use the acceptance–rejection method given in
[Bauemer and Meerschaert(10)], [Kawai and Masuda(11)].

remind: $(\widehat{\sigma_n^i})^2 = \frac{1}{\Delta_n K} \sum_{l=1}^{K} \left(\Delta_{i-l}^n X\right)^2$

		$a_n^i = 5$			$a_n^i = 5 \times \widehat{\sigma_n^i}$			$a_n^i = 5 \times \widetilde{\sigma_n^i}$		
t_n	n	mean	std dev	j filt	mean	std dev	j filt	mean	std dev	j filt
10	100	1.69	0.386	5.16	1.87	0.455	1.19	1.86	0.458	1.03
	400	1.91	0.329	9.78	2.00	0.400	4.56	1.98	0.362	5.73
	1000	1.98	0.262	13.7	2.03	0.410	8.59	2.01	0.341	10.5
50	500	1.66	0.150	27.0	1.82	0.184	7.62	1.81	0.184	9.07
	2000	1.85	0.105	49.5	1.96	0.177	23.3	1.94	0.104	31.3
	5000	1.92	0.088	71.1	1.99	0.166	43.4	1.97	0.088	55.3
100	3000	1.81	0.079	52.8	1.94	0.129	38.6	1.92	0.070	52.8
	10^4	1.91	0.064	142	1.98	0.110	86.7	1.97	0.056	111
	3×10^4	1.96	0.056	213	1.99	0.083	165	1.99	0.046	200

Table : Monte Carlo estimates of mean and standard deviation from 2000 samples of $\hat{\theta}_{1,n}^{\mathrm{OU}}$ for an OU process with tempered α-stable jumps, $\alpha = 0.9$, $\sigma = 1$, $b = 10^{-2}$, and true parameter $\theta_1 = 2$.

$\widehat{\sigma}_n^i$ tends to overestimate σ due to the presence of the infinite number of jumps of the stable process.

We propose to reduce the contribution of the stable process in the estimation of the local volatility by removing in the sum $(\widehat{\sigma_n^i})^2 = \frac{1}{\Delta_n K} \sum_{l=1}^{K} \left(\Delta_{i-l}^n X\right)^2$ the contribution of the biggest increment $\max_{l \in \{i-K,\ldots,i-1\}} |\Delta_l^n X|^2$.

This tends to suppress the contribution of the largest jump of the stable process and considerably reduces the upward bias for the estimation of the local volatility. We note $\widetilde{\sigma}_n^i$ this correction of the quantity $\widehat{\sigma}_n^i$

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Conclusion

Our work shows that by focusing on the drift estimation the condition $n\Delta_n^2 \to 0$ can be relaxed.

It is in accordance with the condition $n\Delta_n^3 \to 0$ of [Florens-Zimrou(89)] and [Yoshida(92)] in the case of drift estimation for continuous diffusions.

It seems preferable to filter less jumps than the true number of jumps=small jumps can be tolerated

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Thank you for your attention!