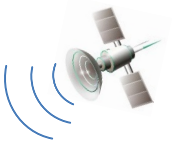
Application of the local Markov approximation method for the statistical analysis of quasi-deterministic and random signals with unknown discontinuous parameters



By

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Abstract

- Statistical analysis algorithms of information processes with unknown discontinuous parameters

- Quasi-deterministic or Gaussian signals against Gaussian distortions

- Approach in practical applications for the analysis of detectors and measurers

- Local Markov approximation method, maximum likelihood method

Introduction

Receiving the information signals with unknown discontinuous parameters:

$$x(t)=\gamma\, s\left(t,\vec{\vartheta}_0\right)+n(t) \longrightarrow \boxed{\begin{array}{c}\textbf{Optimal}\\ \textbf{receiver}\end{array}} \longrightarrow L\left(\vec{\vartheta}\right),\quad \vec{\vartheta}\in\Theta\,,\quad \gamma=0 \text{ or } 1$$

where

$x(t)$ – realization of the observable data,

$s\left(t,\vec{\vartheta}_0\right)$ – useful signal,

$n(t)$ – Gaussian noise,

$\vec{\vartheta}_0$ – true value of unknown vector parameter,

$L\left(\vec{\vartheta}\right)$ – decision statistics (logarithm of the functional of likelihood ratio),

$\vec{\vartheta}$ – current value of unknown parameter,

Θ – area of parameter possible values.

Detection problem

$$\sup L\left(\vec{\vartheta}\right) \underset{0}{\overset{1}{\gtrless}} c$$

Estimation problem

$$\vec{\vartheta}_m = \arg\sup L\left(\vec{\vartheta}\right)$$

where '*c*' is where threshold defined by set optimality criterion

We consider the one-dimensional parameter below for simplicity: $\vec{\vartheta} = l, \quad l \in [L_1, L_2]$

Characteristics of the processing algorithm efficiency

For detection algorithm:

$\alpha = P\left[\sup L(l) > c \middle| x(t) = n(t)\right]$ – false-alarm probability

$\beta = P\left[\sup L(l) < c \middle| x(t) = s(t, l_0) + n(t)\right]$ – missing signal probability

For estimation algorithm:

$b\left(l_m | l_0\right) = \langle l_m - l_0 \rangle$ – conditional bias (systematic error)

$V\left(l_m | l_0\right) = \left\langle \left(l_m - l_0\right)^2 \right\rangle$ – conditional variance (error mean square)

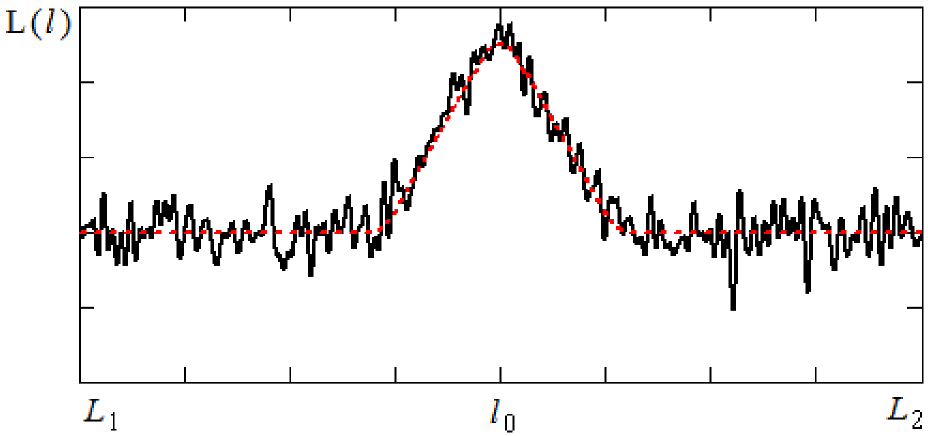
Definition of characteristics of signal detection and discontinuous parameter estimation by the local Markov approximation method

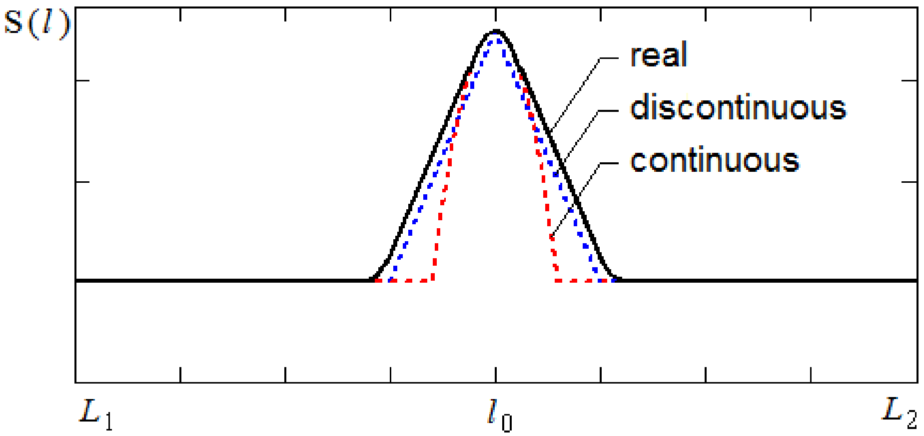
$s(t, l_0)$ – quasi-determined or random (Gaussian) function

The logarithm of the functional of the likelihood ratio is Gaussian (or asymptotically Gaussian) random process

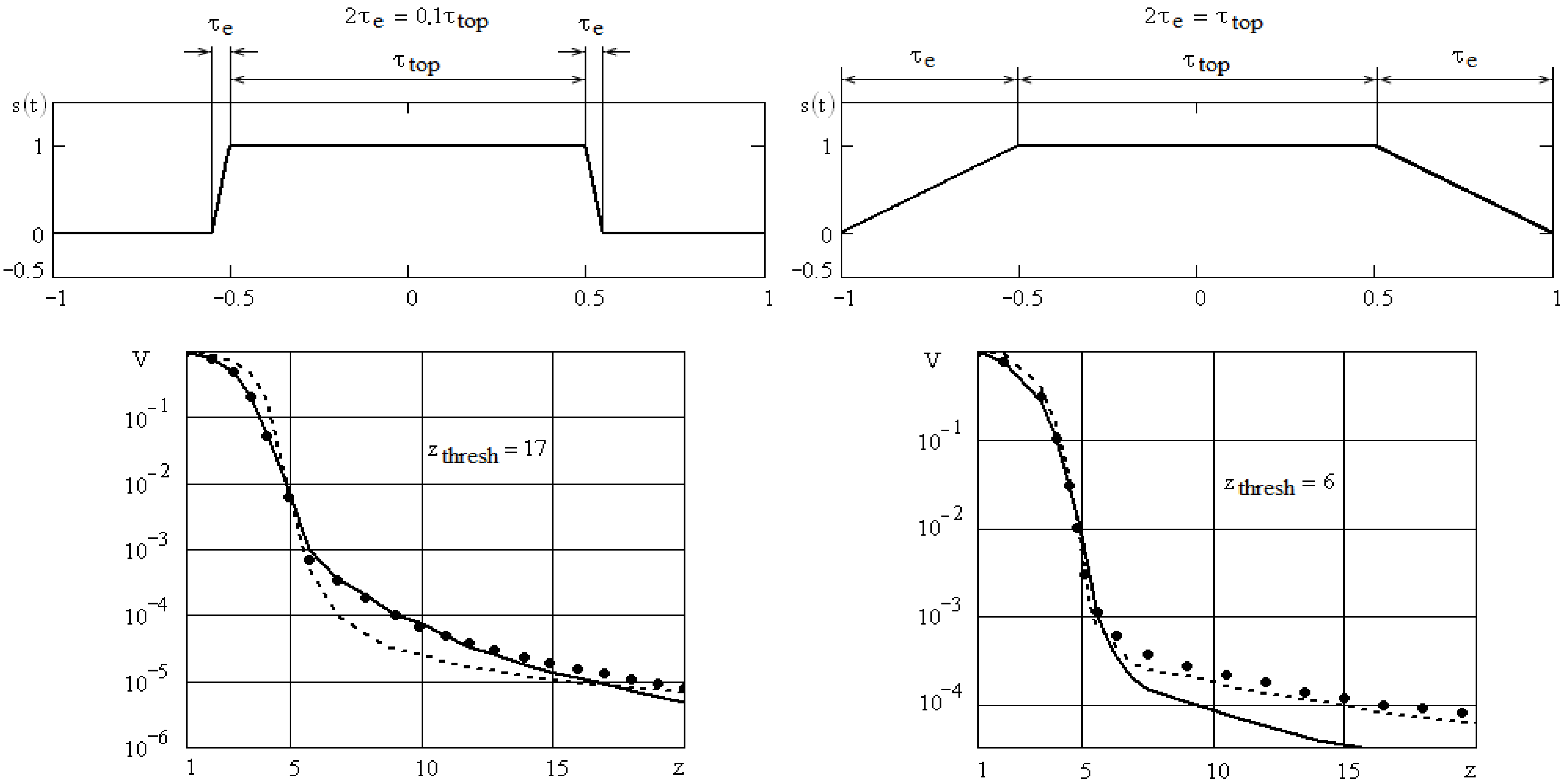
$$L(l) = S(l) + N(l)$$

where $S(l) = \langle M(l) \rangle$, $N(l) = M(l) - \langle M(l) \rangle$





Output signal-to-noise ratio $z^2 = S^2(l_0) / \langle N^2(l_0) \rangle \underset{\text{discontinuous}}{\overset{\text{continuous}}{\gtrless}} z^2_{\text{threshold}}$



variance of the discontinuous estimate of the time of arrival

variance of the continuous estimate of the time of arrival

The signal component has the following exact or asymptotic (with increasing the signal-to-noise ratio) form:

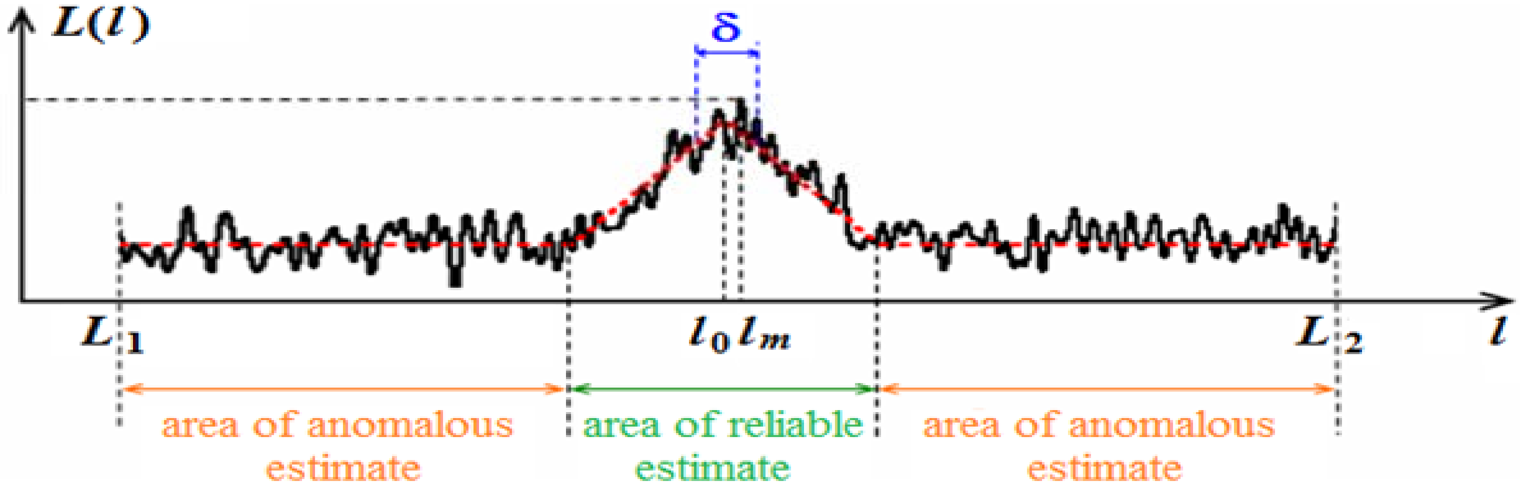
$$S(l) = S_0 \max\left(0, 1 - |l - l_0|\right) + S_N$$

The noise component is centered Gaussian or asymptotically (with increasing the signal-to-noise ratio) Gaussian random process with correlation function

$$B(l_1, l_2) = \langle N(l_1) N(l_2) \rangle = \sigma_N^2 \max\left(0, 1 - |l_1 - l_2|\right) + \left(\sigma_S^2 - \sigma_N^2\right) \times$$
$$\times \max\left[0, 1 + \min\left(0, l_1 - l_0, l_2 - l_0\right) - \max\left(0, l_1 - l_0, l_2 - l_0\right)\right],$$

σ_S^2 and σ_N^2 are the dispersions of the process $N(l)$, if $|l - l_0| < 1$ and $|l - l_0| \geq 1$, correspondingly

Areas of an interval of possible values of the parameter l_o

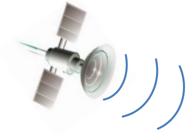


$$\Gamma_N = [L_1, l_0 - 1) \cup (l_0 + 1, L_2] \qquad \Gamma_S = [l_0 - 1, l_0 + 1] \qquad \Gamma_N = [L_1, l_0 - 1) \cup (l_0 + 1, L_2]$$

To start with we consider characteristics of the estimate $l_m \xrightarrow[z \to \infty]{\text{in mean square}} l_0$

where $z^2 = \left[S(l_0) - S_N\right]^2 \big/ \left\langle N^2(l_0) \right\rangle = S_0^2 / \sigma_S^2$ is signal-to-noise ratio.

For the definition of the characteristics of the reliable estimate l_m under the condition $z^2 >> 1$:



The functional $L(l)$ in the small region $[l_0 - \delta, l_0 + \delta]$ of point $l = l_0$

- $L(l)$ is not Markov random process
- its decrement $\Delta(l) = \left[M(l) - M(x)\right] / \sigma_S$ has the covariance function:

$$B_\Delta(l_1, l_2) = \left\langle \left[\Delta(l_1) - \langle \Delta(l_1) \rangle\right] \left[\Delta(l_2) - \langle \Delta(l_2) \rangle\right] \right\rangle =$$
$$= (2-g) \begin{cases} \min(|l_1 - x|, |l_2 - x|), & (l_1 - x)(l_2 - x) \geq 0 , \\ 0 , & (l_1 - x)(l_2 - x) < 0 , \end{cases}$$

This implies:

- realizations of the process $\Delta(l)$ in the intervals $[l_0 - \delta, x)$, $[x, l_0 + \delta]$ are not correlated and therefore they are statistically independent, as being Gaussian (asymptotic Gaussian);

- within each of the intervals , conditions of the Doob's theorem are satisfied, so $\Delta(l)$ is Markov random process of diffusion type. Under $l \geq x$ the drift K_1 and diffusion K_2 coefficients of the process $\Delta(l)$ are determined as:

$$K_1 = \begin{cases} z, & l < l_0, \\ -z, & l \geq l_0, \end{cases} \qquad K_2 = 2 - g \qquad g = \left(\sigma_S^2 - \sigma_N^2\right) / \sigma_S^2 \qquad x \in [l_0 - \delta, l_0 + \delta]$$

Distribution function of the reliable estimate l_m

$$F_0\left(x\middle|l_0\right)=P\left[l_m<x\right]=P\left[\sup_{l<x}\Delta(l)>\sup_{l\ge x}\Delta(l)\right]$$

$$F_2(u,v,x)=P\left[\max_{l<x}\Delta(l)<u,\max_{l\ge x}\Delta(l)<v\right]$$

$$w_2(u,v,x)=\partial^2F_2(u,v,x)/\partial u\partial v$$

Indeed,

$$F_0\left(x\middle|l_0\right)=\int_0^\infty\int_0^u w_2(u,v,x)\ dvdu=\int_0^\infty\left[\frac{\partial F_2(u,v,x)}{\partial u}\right]\Bigg|_{v=u}du$$

Making use of the asymptotical statistical independence of the random process $\Delta(l)$ values in intervals $[l_0-\delta,\mathrm{x})$, $[\mathrm{x},l_0+\delta]$:

$$F_2(u,v,x)=P_{1x}(u)P_{2x}(v)$$

$$P_{1x}(u)=P\left[\sup_{l<x}\Delta(l)<u\right]$$

$$P_{2x}(v)=P\left[\sup_{l\ge x}\Delta(l)<v\right]$$

Then

$$F_0\left(x\middle|l_0\right)=\int_0^\infty P_{2x}(u)dP_{1x}(u)\quad,\quad x\in[l_0-\delta,l_0+\delta]$$

Calculating the probabilities $P_{1x}(u)$ and $P_{2x}(v)$, using Markov properties of the process $\Delta(l)$

For this purpose we introduce the random process $\zeta(l) = v - \Delta(l)$, $v > 0$

Within the interval $l \in [x, l_0 + \delta]$, the probability $P_{2x}(v)$

$$P_{2x}(v) = P\left[\underset{x \le l \le l_0 + \delta}{\Delta(l)} < v\right] = P\left[\underset{x \le l \le l_0 + \delta}{\zeta(l)} > 0\right] = \int_0^\infty w_{2x}(y, l_0 + \delta)\,dy \quad , \quad v > 0$$

Here $w_{2x}(y, l_0 + \delta)$ is the probability density that the process $\zeta(l)$, beginning at the moment $l = x$ from the value $\zeta(x) = v$, will reach the value $\zeta(l_0 + \delta) = y$ by the moment $l = l_0 + \delta$ and, at the same time, over the interval $[x, l_0 + \delta]$ the process $\zeta(l)$ lies within an interval $(0, \infty)$.

The process $\zeta(l)$, as well as $\Delta(l)$, is Markov random process of diffusion type with drift $K_1' = -K_1$ coefficient and diffusion coefficient $K_2' = K_2$

The probability density can be found from the solution of the direct Fokker-Planck-Kolmogorov equation:

$$\frac{\partial w_{2x}(y,l)}{\partial l} - \frac{\partial}{\partial y}\left[K_1 w_{2x}(y,l)\right] - \frac{1}{2}\frac{\partial^2}{\partial y^2}\left[K_2 w_{2x}(y,l)\right] = 0$$

Solving Fokker-Planck-Kolmogorov equation for two cases: 1) $x > l_0$ and 2) $x < l_0$

$$F_0\left(x|l_0\right)=\begin{cases} P_m\left(\left|l_0-x\right|\right), & -\infty \le x < l_0, \\ 1-P_m\left(\left|l_0-x\right|\right), & l_0 \le x \le \infty, \end{cases}$$

$$P_m(v)=\left(\frac{5}{2}+\frac{2z^2v}{2-g}\right)\left[1-\Phi\left(z\sqrt{\frac{v}{2-g}}\right)\right]-\frac{3}{2}\exp\left(\frac{4z^2v}{2-g}\right)\times$$
$$\times\left[1-\Phi\left(3z\sqrt{\frac{v}{2-g}}\right)\right]-\sqrt{\frac{2z^2v}{\pi(2-g)}}\exp\left[-\frac{z^2v}{2(2-g)}\right]$$

or

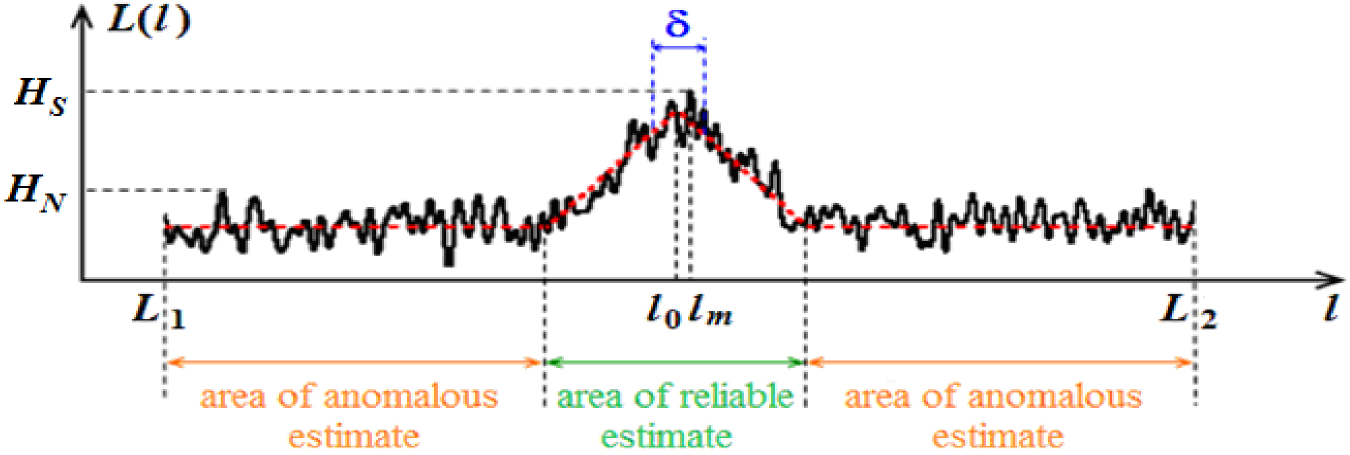
$$w_0\left(x|l_0\right)=\frac{d}{dx}F_0\left(x|l_0\right)=\frac{2z^2}{2-g}\left\{3\exp\left(\frac{4z^2\left|x-l_0\right|}{2-g}\right)\times\right.$$
$$\left.\times\left[1-\Phi\left(3z\sqrt{\frac{\left|x-l_0\right|}{2-g}}\right)\right]+\Phi\left(z\sqrt{\frac{\left|x-l_0\right|}{2-g}}\right)-1\right\},\quad x\in(-\infty,\infty).$$

$$b_0\left(l_m|l_0\right)=\left\langle l_m-l_0\right\rangle=\int_{-\infty}^{\infty}\left(x-l_0\right)w_0\left(x|l_0\right)dx=0$$

$$V_0\left(l_m|l_0\right)=\left\langle\left(l_m-l_0\right)^2\right\rangle=\int_{-\infty}^{\infty}\left(x-l_0\right)^2 w_0\left(x|l_0\right)dx=13(2-g)^2/8z^4$$

Both increases with z.

The probability $P_0 = P\left[\left|l_m - l_0\right| \le 1\right]$ of the reliable estimate l_m



We introduce the functional $\overset{\circ}{L}(l) = L(l) - S_N$ and the random variables

$$H_S = \sup_{l \in \Gamma_S} \overset{\circ}{L}(l) \qquad H_N = \sup_{l \in \Gamma_N} \overset{\circ}{L}(l)$$

Then the probability P_0 : $P_0 = P\left[H_S > H_N\right]$

Distribution functions of random variables

$$F_N(\kappa) = P[H_N/\sigma_N < \kappa] \qquad F_S(\kappa) = P[H_S/\sigma_S < \kappa]$$

Then the probability

$$P_0 = \int F_N(\kappa)\, dF_S(\kappa/r)$$

where $r = \sigma_S/\sigma_N$ and integration is carried out for all possible values κ.

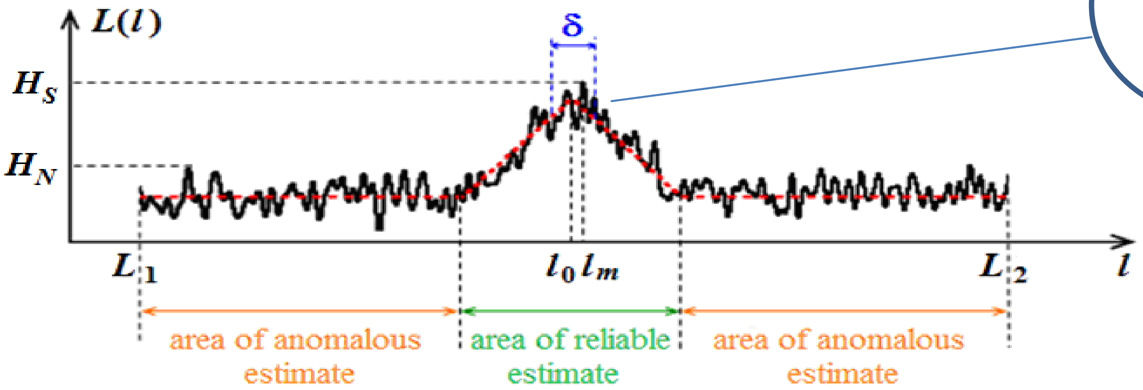
$$\mathrm{F_N}(\kappa) = \mathrm{P}\left[\overset{\circ}{\mathrm{M}}(l)/\sigma_\mathrm{N} < \kappa\right] = \mathrm{P}[\mathrm{N}(l)/\sigma_\mathrm{N} < \kappa], \quad l \in [L_1, l_0 - 1] \cup [l_0 + 1, L_2]$$

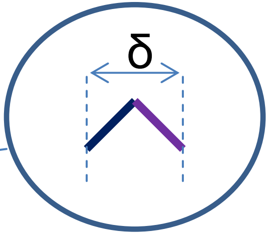
The approximation of the function $F_N(\kappa)$:

$$F_N(\kappa) = \begin{cases} \exp\left[-\dfrac{m\kappa}{\sqrt{2\pi}} \exp\left(-\dfrac{\kappa^2}{2}\right)\right], & \kappa \geq 1, \\ 0, & \kappa < 1. \end{cases}$$

Accuracy of this formula increases with *m* and κ

For the probability $F_S(\kappa)$





$$F_1(\kappa) = P\left[\Delta_0(l)_{l_0-\delta \le l < l_0} < \kappa\right]$$

$$F_2(\kappa) = P\left[\Delta_0(l)_{l_0 \le l < l_0+\delta} < \kappa\right]$$

The values are statistically independent

$$F_S(\kappa) = P\left[\Delta_0(l) < \kappa - \kappa_0\right], \quad l \in [l_0 - \delta, l_0 + \delta]$$

Here $\Delta_0(l) = \Delta(l)\big|_{x=l_0} = \left[L(l) - L(l_0)\right]/\sigma_S$ and $\kappa_0 = \left[L(l_0) - S_N\right]/\sigma_S$ is the random values with probability density $w_0(x) = \exp\left[-(x-z)^2/2\right]/\sqrt{2\pi}$

Then

$$F_S(\kappa) = P\left[\Delta_0(l)_{l_0-\delta \le l < l_0} < \kappa - \kappa_0\right] P\left[\Delta_0(l)_{l_0 \le l < l_0+\delta} < \kappa - \kappa_0\right] \quad \text{or} \quad F_S(\kappa) = \int_{-\infty}^{\kappa} F_1(\kappa - y) F_2(\kappa - y) w_0(y)\, dy$$

As a result:

$$F_S(\kappa)=\frac{1}{\sqrt{2\pi}}\int_0^{\infty}\left\{1-\exp\left[-\frac{2zv}{2-g}\right]\right\}^2\exp\left[-\frac{(\kappa-v-z)^2}{2}\right]dv=$$

$$=\Phi(\kappa-z)-2\exp\left[\psi^2z^2/2+\psi z(z-\kappa)\right]\Phi\left[\kappa-z(\psi+1)\right]+$$

$$+\exp\left[2\psi^2z^2+2\psi z(z-\kappa)\right]\Phi\left[\kappa-z(2\psi+1)\right].$$

The probability of the reliable estimate:

$$P_0=\frac{2\psi z}{r}\exp\left(\frac{2\psi^2z^2}{2}+2\psi z^2\right)\int_1^{\infty}\exp\left[-\frac{mx}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)\right]\left\{\exp\left(-\frac{\psi zx}{r}\right)\times\right.$$

$$\left.\times\Phi\left[\frac{x}{r}-z(\psi+1)\right]-\exp\left[\frac{3\psi^2z^2}{2}+\psi z\left(z-\frac{2x}{r}\right)\right]\Phi\left[\frac{x}{r}-z(2\psi+1)\right]\right\}dx.$$

Here: $\psi=2/(2-g)$

The characteristics of the signal parameter estimate l_{m} with anomalous errors :

$$b(l_m|l_0)=P_0b_0(l_m|l_0)+(1-P_0)\left[(L_2+L_1)/2-l_0\right]=(1-P_0)\left[(L_2+L_1)/2-l_0\right]$$

$$V(l_m|l_0)=P_0V_0(l_m|l_0)+(1-P_0)\left[\left(L_2^2+L_1L_2+L_1^2\right)/3-(L_2+L_1)\,l_0+l_0^2\right]$$

Detection characteristics

When the useful signal is absent, the false-alarm probability is taken place.

$$\alpha = \begin{cases} 1-\exp\left[-\left(mu/\sqrt{2\pi}\right)\exp\left(-u^2/2\right)\right], & u \geq 1, \\ 1, & u < 1, \end{cases}$$

When the useful signal is present, the missing signal probability starts to work, if $u \geq 1$,

$$\beta = \exp\left[-\left(mu/\sqrt{2\pi}\right)\exp\left(-u^2/2\right)\right]\left\{\Phi\left(u/r-z\right)-2\exp\left[\psi^2 z^2/2+\psi z\left(z-u/r\right)\right]\times\right.$$

$$\left.\times\Phi\left[u/r-z\left(\psi+1\right)\right]+\exp\left[2\psi^2 z^2+2\psi z\left(z-u/r\right)\right]\Phi\left[u/r-z\left(2\psi+1\right)\right]\right\},$$

and, if $u < 1$, $\beta \approx 0$.

Accuracy of this formula increases with u, m, z.

Where, $u = \left(c - S_N\right)/\sigma_N$

Reception of the Quasi-deterministic Video Pulse with Unknown Time of Arrival

The observable realization:

$$x(t) = s(t,\lambda_0) + n(t) + \nu(t) \ , \quad t \in [0,T] \ , \qquad s(t,\lambda_0) = aI\left(\frac{t-\lambda_0}{\tau}\right), \qquad I(x) = \begin{cases} 1, & |x| \le 1/2 \,, \\ 0, & |x| > 1/2 \,, \end{cases}$$

where

λ_0 – unknown signal time of arrival

a – signal amplitude

$n(t)$ – Gaussian white noise

$\nu(t)$ – correlated distortions, stationary centered Gaussian random process possessing the spectral density

$$G(\omega) = (\gamma/2)\, I(\omega/\Omega)$$

Ω – bandwidth,

γ – value of the spectral density of the process $\nu(t)$

The process $\nu(t)$ fluctuations are "fast", so the following condition is satisfied

$$\mu = \tau\Omega/4\pi >> 1$$

The logarithm of FLR:

$$M(\lambda)=\frac{\gamma}{N_0(N_0+\gamma)}\int_0^T y^2(t)\,dt+\frac{2a}{N_0+\gamma}\int_{\lambda-\tau/2}^{\lambda+\tau/2} x(t)\,dt-\frac{a^2\tau}{N_0+\gamma}-\mu\ln\left(1+\frac{\gamma}{N_0}\right)$$

where $\mu=\tau\Omega/4\pi>>1$, $y(t)=\int_{-\infty}^{\infty}x(t')h(t-t')\,dt$ and $h(t)$ is the function which spectrum $H(\omega)$ satisfies to a condition $|H(\omega)|^2=I(\omega/\Omega)$

The maximum likelihood estimate: $\lambda_m=\underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup}\, M_0(\lambda)$, $M_0(\lambda)=\int_{\lambda-\tau/2}^{\lambda+\tau/2} x(t)\,dt$

$$M_0(\lambda)=S(l)+N(l)\begin{cases} S(l)=\langle M_0(\lambda)\rangle \longrightarrow S(l)=S_0\max(0,1-|l-l_0|)+S_N \\ N(l)=M_0(\lambda)-\langle M_0(\lambda)\rangle \longrightarrow \sigma_S^2=\sigma_N^2=\tau(N_0+\gamma)/2 \end{cases}$$

$$S_0=a\tau$$

$$S_N=0$$

For anomalous error, conditional bias and variance of the maximum likelihood estimate:

$$b(l_m|l_0)=(1-P_0)[(L_2+L_1)/2-l_0]$$

$$V(l_m|l_0)=P_0V_0(l_m|l_0)+(1-P_0)\left[\left(L_2^2+L_1L_2+L_1^2\right)/3-(L_2+L_1)\ l_0+l_0^2\right]$$

For conditional variance and probability of the normalized maximum likelihood reliable estimate:

$$V_0(l_m|l_0)=13/2z^4$$

$$P_0=2z\exp\left(\frac{3z^2}{2}\right)\int_1^\infty \exp\left[-\frac{mx}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)\right]\{\exp(-zx)\times$$
$$\times\Phi(x-2z)-\exp[z(5z-4x)/2]\Phi(x-3z)\}dx$$

For the false-alarm probability : $\alpha=\begin{cases}1-\exp\left[-\left(mu/\sqrt{2\pi}\right)\exp\left(-u^2/2\right)\right], & u\geq 1,\\ 1, & u<1,\end{cases}$

For the missing signal probability, if $u\geq 1$,

$$\beta=\exp\left[-\left(mu/\sqrt{2\pi}\right)\exp\left(-u^2/2\right)\right]\left\{\Phi(u/r-z)-2\exp\left[\psi^2z^2/2+\psi z(z-u/r)\right]\times\right.$$
$$\left.\times\Phi\left[u/r-z(\psi+1)\right]+\exp\left[2\psi^2z^2+2\psi z(z-u/r)\right]\Phi\left[u/r-z(2\psi+1)\right]\right\},$$

and, if $u<1, \beta\approx 0$.

where, $u=(c-S_N)/\sigma_N$ | $\psi=1$ | $L_{1,2}=\Lambda_{1,2}/\tau$

$z^2=2a^2\tau/(N_0+\gamma)$ | $r=1$ | $m=L_2-L_1$

Theoretical and experimental values of detection characteristics and normalized variance of video pulse

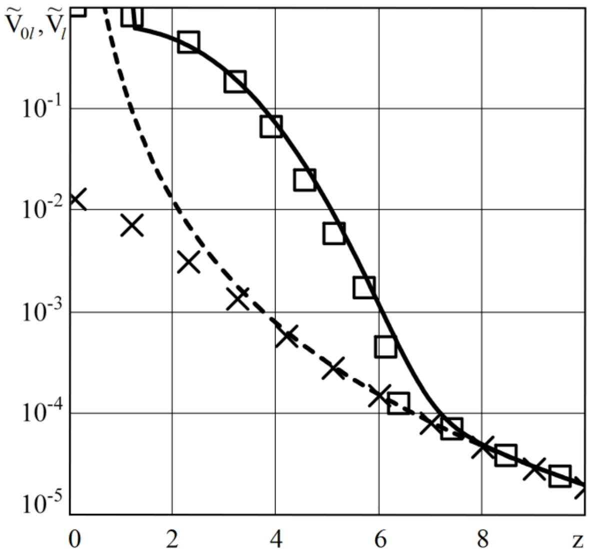


Fig.1. Normalized variance of appearance time estimate

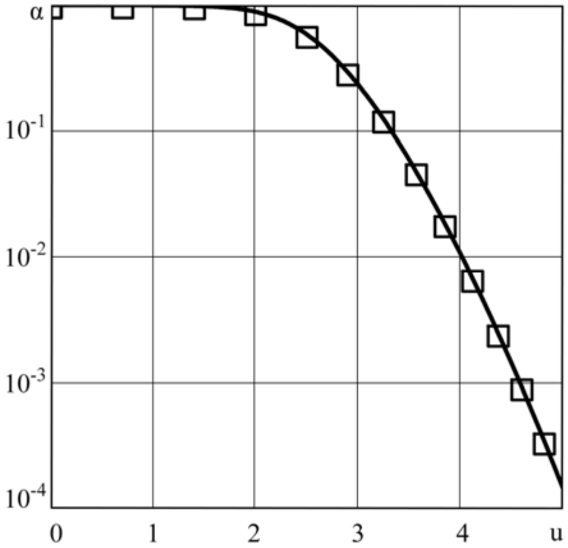


Fig 2. False-alarm probability

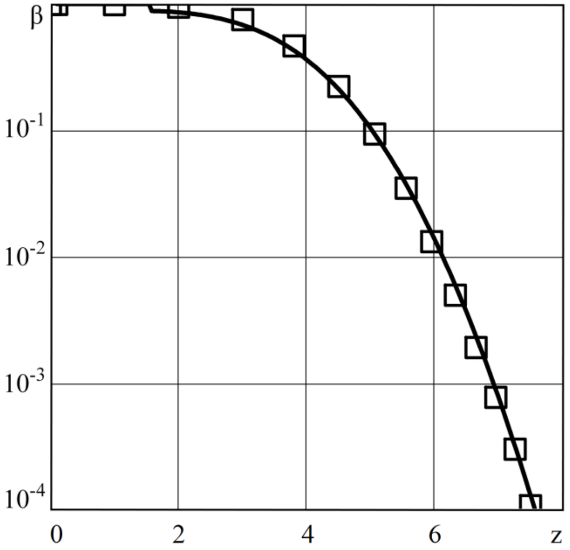


Fig 3. Missing probability

The observable realization:

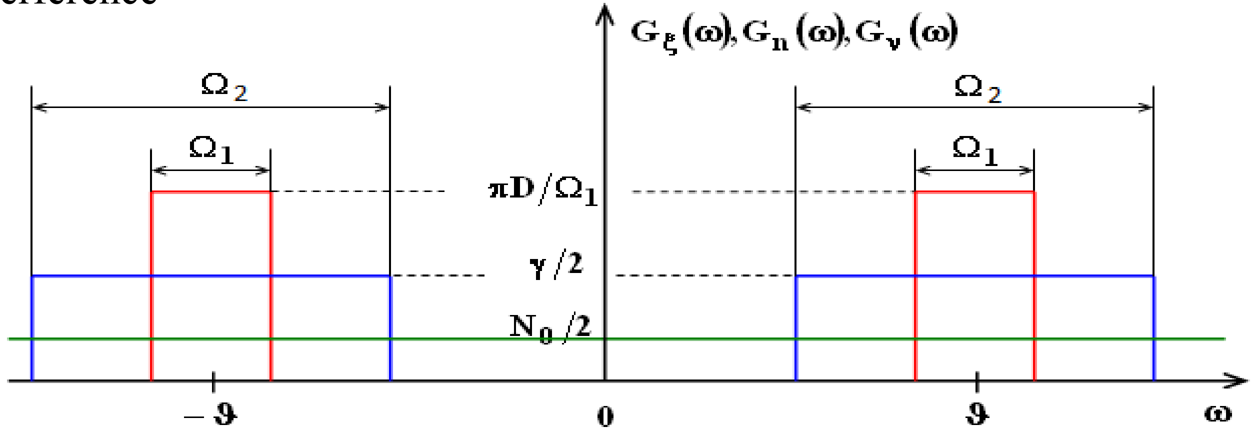
$$x(t) = s(t, \lambda_0) + n(t) + \nu(t) \;, \quad t \in [0, T] \;, \qquad I(x) = \begin{cases} 1, & |x| \le 1/2 \,, \\ 0, & |x| > 1/2 \,, \end{cases}$$

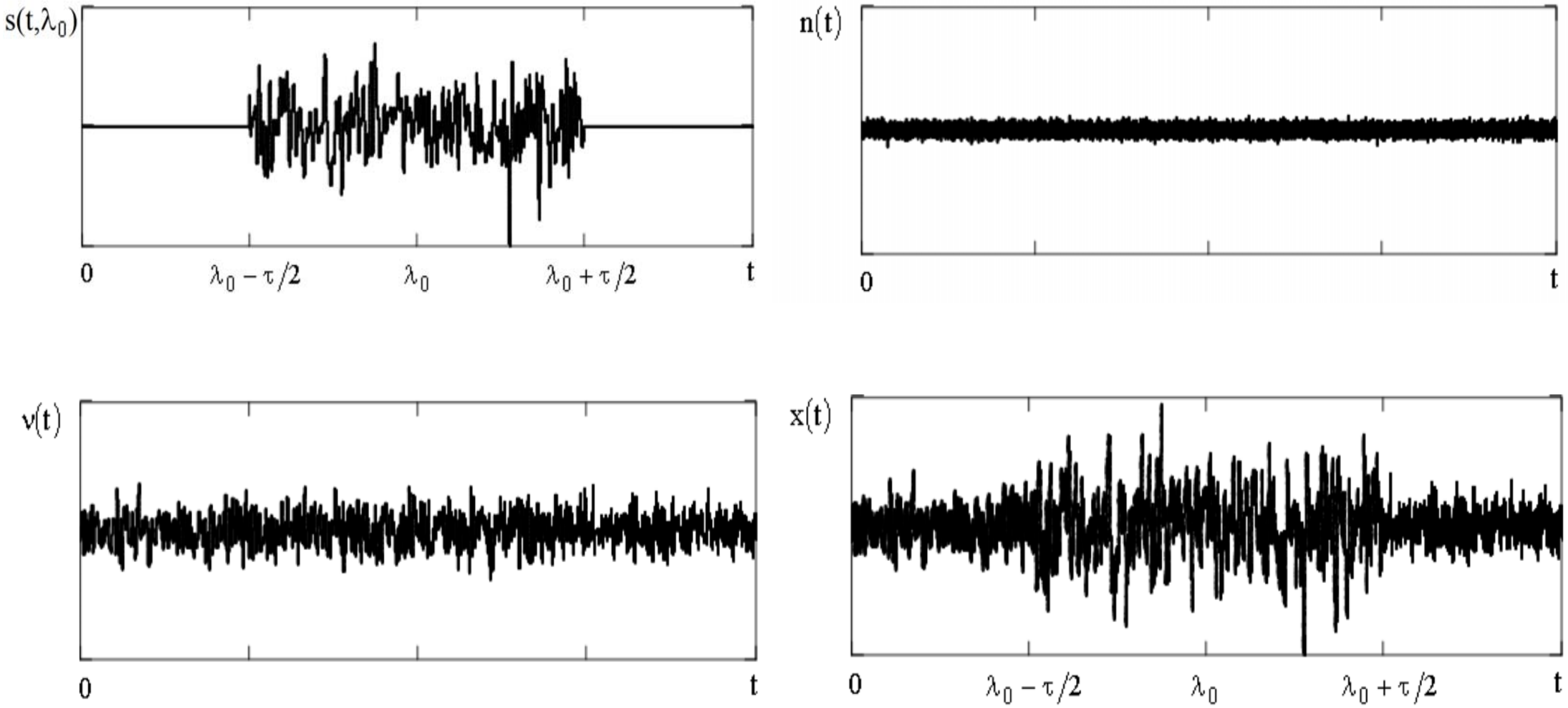
The useful signal of a random radio pulse:

$$s(t, \lambda_0) = \xi(t) \, I[(t - \lambda_0)/\tau]$$

where $\xi(t)$ – stationary centered high-frequency random process, $n(t)$ – Gaussian white noise,

$\nu(t)$ – band Gaussian interference




$s(t,\lambda_0)$
0
$\lambda_0 - \tau/2$
λ_0
$\lambda_0 + \tau/2$
t
$n(t)$
0
t
$\nu(t)$
0
t
$x(t)$
0
$\lambda_0 - \tau/2$
λ_0
$\lambda_0 + \tau/2$
t

If $\mu = \tau\Omega_1/2\pi >> 1$ (fluctuations are "fast"), then the logarithm of FLR takes the form of

$$M(\lambda) = \frac{d\,M_\tau(\lambda)}{(N_0+\gamma)(N_0+\gamma+d)} + \frac{\gamma\,M_T}{N_0\,(N_0+\gamma)} - \mu\left[\ln\left(1+\frac{\gamma+d}{N_0}\right) - (K-1)\ln\left(1+\frac{\gamma}{N_0}\right)\right]$$

where $d = 2\pi D/\Omega_1$, $K = T\Omega_2/\tau\Omega_1$, $M_\tau(\lambda) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} y_1^2(t)\,dt$, $M_T = \int_0^T y_2^2(t)\,dt$, $y_i(t) = \int_{-\infty}^{\infty} x(t')\,h_i(t-t')\,dt'$,

$i = 1,2$ and $h_i(t)$ is the function which spectrum $H_i(\omega)$ satisfies to a condition

$$|H_i(\omega)|^2 = I[(\vartheta-\omega)/\Omega_i] + I[(\vartheta+\omega)/\Omega_i]$$

The maximum likelihood estimate of the time of arrival:

$$\lambda_m = \underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup}\; M(\lambda) = \underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup}\; M_\tau(\lambda)$$

$$M_\tau(\lambda) = S(l) + N(l)$$

$$\mathrm{S}(l) = \langle \mathrm{M}_\tau(\lambda)\rangle \rightarrow S(l) = S_0 \max(0, 1-|l-l_0|) + S_N$$

$$\mathrm{S}_0 = \tau \mathrm{D}$$

$$\mathrm{S}_\mathrm{N} = \tau(\mathrm{E}_\mathrm{N} + \mathrm{E}_\gamma)$$

$$\mathrm{E}_\mathrm{N} = \mathrm{N}_0\Omega_1/2\pi$$

$$\mathrm{E}_\gamma = \gamma\Omega_1/2\pi$$

$$\mathrm{N}(l) = \mathrm{M}_\tau(\lambda) - \langle \mathrm{M}_\tau(\lambda)\rangle \longrightarrow$$

$$\sigma_S^2 = [\tau E_N(1+q_\nu+q)]^2/\mu$$

$$\sigma_N^2 = [\tau E_N(1+q_\nu)]^2/\mu$$

$$\mathrm{q} = \mathrm{D}/\mathrm{E}_\mathrm{N}$$

$$\mathrm{q}_\nu = \gamma/\mathrm{N}_0$$

For anomalous errors, the conditional bias and the variance of the maximum likelihood estimate:

$$b\left(l_m | l_0\right) = \left(1 - P_0\right)\left[\left(L_2 + L_1\right)/2 - l_0\right]$$

$$V\left(l_m | l_0\right) = P_0 V_0\left(l_m | l_0\right) + \left(1 - P_0\right)\left[\left(L_2^2 + L_1 L_2 + L_1^2\right)\Big/3 - \left(L_2 + L_1\right)\, l_0 + l_0^2\right]$$

For the conditional variance and the probability of the normalized maximum likelihood reliable estimate:

$$V_0\left(l_m | l_0\right) = 13\big/2z^4$$

$$P_0 = 2z \exp\left(\frac{3z^2}{2}\right)\int_1^\infty \exp\left[-\frac{mx}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)\right]\left\{\exp\left(-zx\right)\times \times\Phi\left(x - 2z\right) - \exp\left[z\left(5z - 4x\right)/2\right]\Phi\left(x - 3z\right)\right\}dx$$

For the false-alarm probability : $\alpha = \begin{cases} 1 - \exp\left[-\left(mu/\sqrt{2\pi}\right)\exp\left(-u^2/2\right)\right], & u \geq 1, \\ 1, & u < 1, \end{cases}$

For the missing signal probability, if $u \geq 1$, $\beta = \exp\left[-\left(mu/\sqrt{2\pi}\right)\exp\left(-u^2/2\right)\right]\left\{\Phi\left(u/r - z\right) - 2\exp\left[\psi^2 z^2/2 + \psi z\left(z - u/r\right)\right]\times \times\Phi\left[u/r - z\left(\psi + 1\right)\right] + \exp\left[2\psi^2 z^2 + 2\psi z\left(z - u/r\right)\right]\Phi\left[u/r - z\left(2\psi + 1\right)\right]\right\}$,

and, if $u < 1$, $\beta \approx 0$.

where $u = \left(c - S_N\right)/\sigma_N$ | $\psi = \dfrac{2\left(1 + q_\nu + q\right)^2}{\left(1 + q_\nu\right)^2 + \left(1 + q_\nu + q\right)^2}$ | $m = L_2 - L_1$

$z^2 = \mu_1 q^2 / \left(1 + q_\nu + q\right)^2$ | $r = \left(1 + q_\nu + q\right)/\left(1 + q_\nu\right)$ | $L_{1,2} = \Lambda_{1,2}/\tau$

Theoretical and experimental values of detection characteristics and normalized variance of random radio pulse

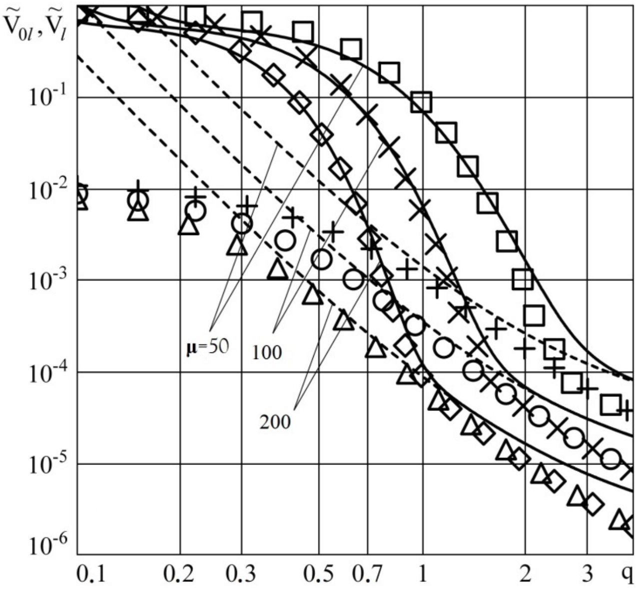


Fig. 4. Normalized variance of appearance time estimate

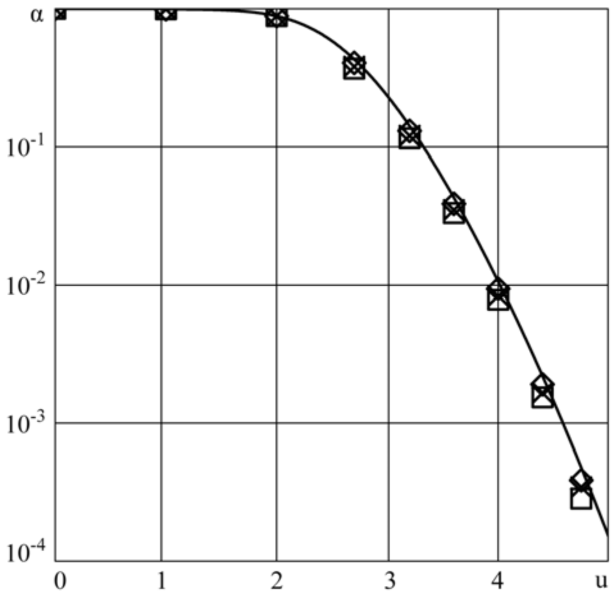


Fig. 5. False-alarm probability

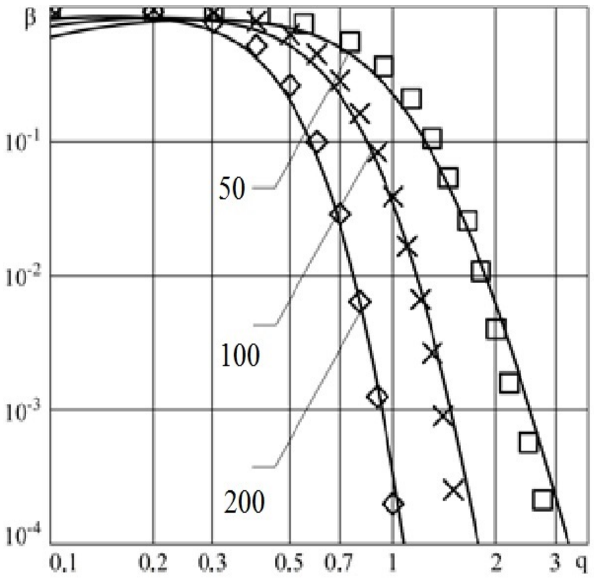


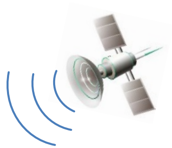
Fig. 6. Missing probability

Conclusion

- For operating efficiency definition of the optimal (maximum-likelihood) receiving devices of the signals with unknown discontinuous parameters a method based on approximation of the solving statistics increments by Markov random process (local Markov approximation method) can be applied.

- With the help of the introduced approach the closed analytical expressions for the characteristic can be found of detectors and measurers of discontinuous quasi-determined and Gaussian random signals, which well describe corresponding experimental data in a wide range of output signal-to-noise ratios.

- The obtained results make it possible theoretically to estimate practical application appropriateness of one or another processing algorithm of discontinuous signals in each specific case.



THANK YOU VERY MUCH

FOR YOUR ATTENTION!